

NLO corrections to lepton pair production beyond the Standard Model at hadron colliders

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ABSTRACT: We consider lepton pair production at a hadron collider in a class of effective theories with the relevant operators being four-fermion contact interaction. Despite the nonrenormalizable nature of the interaction, we explicitly demonstrate that calculating QCD corrections is both possible and meaningful. Calculating the corrections for various differential distributions, we show that these can be substantial and significantly different from those within the SM. Furthermore, the corrections have a very distinctive flavour dependence. And finally, the scale dependence of the cross sections are greatly reduced once the NLO corrections are taken into account.

KEYWORDS: NLO Computations, Beyond Standard Model, Hadronic Colliders, QCD.

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1. Introduction

While the Standard Model (SM) remains a very consistent explanation for nearly all data pertaining to high energy physics experiments, a few small discrepancies persist. Furthermore, there are theoretical issues that cannot even be addressed within the framework of the SM alone. Examples include the replication of the fermion families, the naturalness problem associated with the Higgs scale, charge quantization, the baryon asymmetry in the universe, the presence of dark matter etc.. Clearly, an answer to such vital questions may be obtained only in a model much more ambitious than the SM. Candidates for the role include, amongst others, supersymmetry [1], grand unification [2, 3] (with or without supersymmetry), family symmetries (gauged or otherwise) and compositeness for quarks and leptons [4]. In general, each such scenario (with its peculiar strengths and weaknesses) is associated with an individual set of tell-tale signatures. On the other hand, if the SM is to be a valid effective low-energy description of such bigger structures, one should be able to construct, within the ambit of the SM, operators that would encapsulate a class of remnant effects that could pertain to any of these scenarios. We illustrate this explicitly in the context of one of the above-mentioned scenarios.

The replication of fermion families suggests the possibility of quark-lepton compositeness. In such theories, the fundamental constituents, very often termed *preons* [5], experience an hitherto unknown force on account of an asymptotically free but confining gauge interaction [6]. At a characteristic scale Λ , this interaction would become very strong leading to bound states (composites) which are to be identified as quarks and leptons. In most such models [7, 8], quarks and leptons share at least some common constituents. Since the confining force mediates interactions between such constituents, it stands to reason that

these, in turn, would lead to interactions between quarks and leptons that go beyond those existing within the SM. Well below the scale Λ , such interactions would likely be manifested through an effective four fermion contact interaction [9] term that is an invariant under the SM gauge group. A convenient and general parametrization of such interactions is given by [4, 10]

$$\mathcal{L} = \frac{4\pi}{\Lambda^2} \left[\eta_{ij} (\bar{q} \gamma^\mu P_i q) (\bar{l} \gamma_\mu P_j l) + \xi_{ij} (\bar{q} P_i q) (\bar{l} P_j l) \right], \quad (1.1)$$

where $i, j = L, R$ and P_i are the chirality projection operators. Note that the lagrangian of eq. (1.1) is by no means a comprehensive one and similar operators involving the quarks alone (or the leptons alone) would also exist. However, for our purpose, it would suffice to consider only eq. (1.1). Within this limited sphere of applicability, the strength of the interaction may be entirely absorbed in the scale Λ , and the couplings η_{ij} and ξ_{ij} canonically normalized to ± 1 .

While we have sought to motivate eq. (1.1) in the context of compositeness, these are by no means the only scenarios ones that can give rise to such an effective interaction lagrangian. As is well known, a four-fermion process mediated by a particle with a mass significantly higher than the energy transfer can be well approximated by a contact interaction [9] term with a generic form as in eq. (1.1). Examples include theories with extended gauge sectors, leptoquarks [11], sfermion exchange in a supersymmetric theory with broken R -parity [12] etc.. In all such cases, on integrating out fields with masses $M_i \gtrsim \Lambda$ [13], a series of such higher-dimensional terms obtain. Those in eq. (1.1) are just the lowest order (in Λ^{-1}) ones.

Several points are in order here

- In general, integrating out the heavy fields would result in an almost infinite number of higher-dimensional operators. The terms in eq. (1.1) are just some of the lowest order (in Λ^{-1}) ones relevant to four-fermion processes.
- For a given model, the couplings η_{ij} and ξ_{ij} generated by the process of integrating out heavy fields would be related to each other. Such relations are model-specific and determined, to a large extent, by the flavour structure of the parent theory. As already indicated, we shall not consider any such flavour structure, but hold $\eta, \xi = \pm 1$.
- Even a requirement such as $SU(2) \otimes U(1)$ invariance for the effective lagrangian would imply a relation between such terms, but involving different fields. However, we shall concern ourselves with only those involving $q\bar{q}\ell^+\ell^-$, noting that the results would be essentially the same for its $SU(2)$ cousins.
- Low energy observables (for example, meson decays) lead to severe constraints [14] on several of these couplings and even more so, on their products. Although many of these bounds were derived in the context of specific ultraviolet completions, it is easy to see that they are equally applicable to the generic contact interactions.

- Note that the vector-axial vector (VA) or scalar-pseudoscalar (SP) nature of the η - and ξ -couplings do not necessarily reflect the spin of the integrated out field that led to such terms.

Clearly, operators such as these could, in principle, lead to significant phenomenological consequences in collider experiments, whether e^+e^- [18], eP [17] or hadronic. Given the higher-dimensional nature of \mathcal{L} , it is obvious that the consequent effects would be more pronounced at higher energies. In other words, the fractional deviation over the SM expectations would be concentrated more at higher invariant masses M [15], with possibly some nontrivial dependence on the rapidity y as well [16]. For example, composite quarks and electrons have been proposed as a possible explanation for the high- Q^2 anomaly at HERA [17]. Some of the best constraints on compositeness, for example, came from the OPAL [18] and CDF [19] experiments. More recently the measurement of the Drell-Yan cross section [20] at high invariant masses set the most stringent limits on contact interactions of the type given in eq. (1.1). For example, within the VA -type interaction scenario, the scale Λ is constrained to be $\Lambda \gtrsim 3.3\text{--}6.1$ TeV [21, 22], with the bound depending on the chirality structure of the operator.

As is well known, QCD corrections can alter quite significantly the cross sections at a hadronic collider. Thus, these may have serious bearing on the discovery potential of such experiments. Even for as simple a process as Drell-Yan, the leading order (LO) results seriously underestimate the cross sections. This has led to the incorporation of the next-to-leading order (NLO) or next-to-leading log (NLL) [24, 25] results in Monte Carlo codes [24] or event generators such as JETRAD [23]. However, no calculations exist for the higher order QCD corrections to cross sections mediated by a generic contact interaction. Consequently, all extant collider studies of contact interaction have either been based on just the tree level calculations, or, in some cases, have implicitly assumed that the higher order corrections are exactly the same as in the SM. Clearly, this is an unsatisfactory state of affairs and, in this paper, we aim to rectify this by calculating the next-to-leading order QCD corrections for both the VA -type and the SP -type contact interactions.

It might be argued that, such theories being nonrenormalizable, any higher-loop calculation is fraught with danger. However, the very structure of such terms (namely the current-current form of the lagrangian) along with the fact that only one of the currents comprises coloured fields allows us to reliably calculate QCD corrections. This holds not only for the specific interaction in question, but also for other theories that satisfy the abovementioned criterion [26]. On the other hand, were we to attempt to calculate the NLO electroweak corrections, it is by no means certain that similar levels of reliability or usefulness could be reached.

The rest of the article is organised as follows. In section 2, we start by outlining the general methodology and follow it up with the explicit calculation of the NLO corrections to the differential distribution in the dilepton invariant mass. In the following section, we consider the rapidity distributions. Section 4 contains our numerical results. And finally, we summarize in section 5.

2. NLO corrections

We consider lepton pair production at a hadron collider in the context of a generic contact interaction as exemplified by eq. (1.1). In other words, the process is

$$P(p_1) + \bar{P}(p_2) \rightarrow l^+(l_1) + l^-(l_2) + X(p_X) \quad (2.1)$$

where p_i denote the momenta of the incoming hadrons and l_i those for the outgoing leptons. Similarly, the inclusive hadronic state denoted by X carries momentum p_X . The hadronic cross section is defined in terms of the partonic cross sections convoluted with the appropriate parton distribution functions $f_a^P(x)$ and is given by

$$2S \frac{d\sigma}{dQ^2}{}^{P_1 P_2} = \sum_{ab=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2 f_a^{P_1}(x_1) f_b^{P_2}(x_2) \int_0^1 dz 2\hat{s} \frac{d\sigma^{ab}}{dQ^2} \delta(\tau - zx_1 x_2) \quad (2.2)$$

with x_i being the fraction of the initial state hadron's momentum carried by the parton in question. In other words, the parton momenta k_i are given by $k_i = x_i p_i$. The other variables are defined as

$$\begin{aligned} S &\equiv (p_1 + p_2)^2 & \hat{s} &\equiv (k_1 + k_2)^2 & Q^2 &\equiv (l_1 + l_2)^2 \\ \tau &\equiv \frac{Q^2}{S} & z &\equiv \frac{Q^2}{\hat{s}} & \tau &\equiv z x_1 x_2 . \end{aligned} \quad (2.3)$$

It is convenient to symbolically cast the matrix element for the process as a sum of several current-current pieces with a ‘‘propagator’’ in between. In other words,

$$\mathcal{M}^{\text{Total}} = \sum_j \mathcal{J}_j^{\text{Had}} \cdot P_j \cdot \mathcal{J}_j^{\text{Lept}} \quad (2.4)$$

where the dots (\cdot) denote Lorentz index contractions as appropriate and the propagators P_j are

$$\begin{aligned} P_\gamma &= \frac{i}{Q^2} g_{\mu\nu} \equiv g_{\mu\nu} \tilde{P}_\gamma & P_Z &= \frac{i g_{\mu\nu}}{Q^2 - M_Z^2 - i M_Z \Gamma_Z} \equiv g_{\mu\nu} \tilde{P}_Z \\ P_{VA} &= \frac{4\pi}{\Lambda^2} \equiv \tilde{P}_{VA} & P_{SP} &= \frac{4\pi}{\Lambda^2} \equiv \tilde{P}_{SP} . \end{aligned} \quad (2.5)$$

With this definition, the partonic cross section for the process $a(k_1) + b(k_2) \rightarrow j(q) + \sum_i^m X(p_i)$ is given by

$$2\hat{s} \frac{d\sigma^{ab}}{dQ^2} = \frac{1}{2\pi} \sum_{jj'=\gamma,Z,VA,SP} \int dPS_{m+1} |\mathcal{M}^{ab \rightarrow jj'}|^2 \cdot P_j(Q^2) \cdot P_{j'}^*(Q^2) \cdot \mathcal{L}^{jj' \rightarrow ll'} , \quad (2.6)$$

where $|\mathcal{M}^{ab \rightarrow jj'}|^2$ denotes the square of the hadronic current, and dPS_{m+1} the $(m+1)$ -body phase space element, viz.

$$\begin{aligned} dPS_{m+1} &= \int \prod_i^m \left(\frac{d^n p_i}{(2\pi)^n} 2\pi \delta^+(p_i^2) \right) \frac{d^n q}{(2\pi)^n} 2\pi \delta^+(q^2 - Q^2) \\ &\quad \times (2\pi)^n \delta^{(n)} \left(k_1 + k_2 - q - \sum_i^m p_i \right) \end{aligned} \quad (2.7)$$

where n is the dimension of spacetime and $\delta^+(x)$ carries its usual meaning. The leptonic tensor, given by

$$\mathcal{L}^{jj' \rightarrow ll'} = \int \prod_i^2 \left(\frac{d^n l_i}{(2\pi)^n} 2\pi \delta^+(l_i^2) \right) (2\pi)^n \delta^{(n)}(q - l_1 - l_2) |\mathcal{M}^{jj' \rightarrow l^+ l^-}|^2, \quad (2.8)$$

is straightforward to compute and leads to

$$\mathcal{L}_{jj' \rightarrow ll'} = \begin{cases} \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2} \right) \mathcal{L}_{jj'}(Q^2) & (j, j' = \gamma, Z, VA) \\ \mathcal{L}_{SP}(Q^2) & (j = j' = SP) \end{cases} \quad (2.9)$$

with

$$\begin{aligned} \mathcal{L}_{\gamma\gamma}(Q^2) &= \frac{2\alpha}{3} Q^2, & \mathcal{L}_{ZZ}(Q^2) &= \frac{\alpha}{3} \left((g_l^R)^2 + (g_l^L)^2 \right) Q^2 \\ \mathcal{L}_{\gamma Z}(Q^2) &= \frac{-\alpha g_\ell^V}{6} Q^2, & \mathcal{L}_{SP}(Q^2) &= Q^2 \end{aligned} \quad (2.10)$$

In the above, α denotes the fine structure constant, while $g_a^{L,R}$ parametrize the couplings of the left- and right-chiral fermionic fields to the Z , viz.

$$g_a^V = \frac{1}{2} (g_a^R + g_a^L), \quad g_a^L = -e_a \tan \theta_W, \quad g_a^R = -2T_a^3 \csc 2\theta_W - e_a \tan \theta_W \quad (2.11)$$

in terms of the Weinberg angle (θ_W) and the electric charge (e_a) of the fermion in question.

On substituting for $\mathcal{L}_{jj' \rightarrow ll'}$ in eq. (2.6), we have, for the hadronic cross section,

$$2S \frac{d\sigma^{P_1 P_2}}{dQ^2}(\tau, Q^2) = \frac{1}{2\pi} \sum_{j, j' = \gamma, Z, VA, SP} \tilde{P}_j(Q^2) \tilde{P}_{j'}^*(Q^2) \mathcal{L}_{jj'}(Q^2) W_{jj'}^{P_1 P_2}(\tau, Q^2) \quad (2.12)$$

where the hadronic structure function W is defined to be

$$W_{jj'}^{P_1 P_2}(\tau, Q^2) = \sum_{a, b, j, j'} \int_0^1 dx_1 \int_0^1 dx_2 f_a^{P_1}(x_1) f_b^{P_2}(x_2) \int_0^1 dz \delta(\tau - zx_1 x_2) \bar{\Delta}_{ab}^{jj'}(z, Q^2, \epsilon). \quad (2.13)$$

All that remains is to calculate the bare partonic coefficient function $\bar{\Delta}$:

$$\bar{\Delta}_{ab}^{jj'}(z, Q^2, \epsilon) = \int dPS_{m+1} |\mathcal{M}^{ab \rightarrow jj'}|^2 T_{jj'}(q). \quad (2.14)$$

where $T_{jj'}$ depends upon the spin of the current in question, viz

$$\begin{aligned} T_{jj'}(q) &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2} \right) & (j, j' = \gamma, ZZ, VA) \\ T_{SP}(q) &= 1 \end{aligned} \quad (2.15)$$

To compute the Q^2 distribution of the dilepton pair, one then has to calculate the square of the hadronic matrix element $|\mathcal{M}^{ab \rightarrow jj'}|^2 T_{jj'}(q)$, preferably in a suitable frame so as to render the integrations over the phase space and z easy.

Note that, the bare partonic coefficient function $\bar{\Delta}$ is a singular object, suffering from each of ultraviolet, soft and collinear divergences. To handle these, we adopt dimensional regularisation. The renormalization procedure for the VA -type interactions is quite established and may be found, for example, in ref. [26]. Note that, for the SP type interaction, one-loop corrections results in an extra term — proportional to $\ln(Q^2/\mu^2)$ — as compared to the VA interactions [29]. This, of course, is not unexpected, as contrary to the usual conserved vector currents, a scalar current is renormalized by QCD interactions. It is easy to see that this extra term is precisely the one that is absorbed into the bare contact interaction coupling constant in defining the renormalized coupling ξ_{ij} .

To the ultraviolet regularized (and renormalized) operator, we must add the contribution from the real emission diagrams (gluon bremsstrahlung as well as the Compton process), and this exercise leaves us with a quantity that suffers only from collinear singularities. The latter, of course, can be removed through mass factorization. If μ_F be the factorization scale, then Drell-Yan coefficient functions, after mass factorization by $\Delta_{ab}^{jj'}$, are related to the bare functions through

$$\bar{\Delta}_{ab}^{jj'}(z, Q^2, \epsilon) = \sum_{c,d} \Gamma_{ca}(z, \mu_F^2, 1/\epsilon) \otimes \Gamma_{db}(z, \mu_F^2, 1/\epsilon) \otimes \Delta_{cd}^{jj'}(z, Q^2, \mu_F^2) \quad (2.16)$$

with the convolution defined to be

$$f \otimes g(x) = \int_x^1 \frac{dy}{y} f(y) g\left(\frac{x}{y}\right). \quad (2.17)$$

The kernels Γ_{ab} are related to the leading order Altarelli-Parisi splitting functions [27] $P_{ab}^{(0)}(z)$ through

$$\begin{aligned} \Gamma_{ab}(z, \mu_F^2, 1/\epsilon) &= \delta_{ab} \delta(1-z) + \frac{\alpha_s(\mu^2)}{4\pi\epsilon} \Gamma_{ab}^{(1)}(z, \mu_F^2) \\ &= \delta_{ab} \delta(1-z) + \frac{a_s}{\epsilon} \left(\frac{\mu_F^2}{\mu^2}\right)^{\epsilon/2} P_{ab}^{(0)}(z) \\ a_s &= \frac{\alpha_s(\mu^2)}{4\pi} \end{aligned} \quad (2.18)$$

Expanding eq. (2.16) to order a_s we have

$$\begin{aligned} \bar{\Delta}_{q\bar{q}}^{jj'} &= \Delta_{q\bar{q}}^{(0),jj'} + a_s \frac{2}{\epsilon} \Gamma_{q\bar{q}}^{(1)} \otimes \Delta_{q\bar{q}}^{(0),jj'} + a_s \Delta_{q\bar{q}}^{(1),jj'} \\ \bar{\Delta}_{qg}^{jj'} &= a_s \frac{2}{\epsilon} \Gamma_{qg}^{(1)} \otimes \Delta_{qg}^{(0),jj'} + a_s \Delta_{qg}^{(1),jj'} \end{aligned} \quad (2.19)$$

thereby leading us to the finite coefficient functions. The physical hadronic cross section may be obtained by folding these finite coefficient functions with appropriate parton distribution functions. For the sake of completeness, we present the results below. To begin with, we denote the renormalized parton-parton fluxes by $H_{ab}(x_1, x_2, \mu_F^2)$ where

$$\begin{aligned} H_{q\bar{q}}(x_1, x_2, \mu_F^2) &= f_q^{P_1}(x_1, \mu_F^2) f_{\bar{q}}^{P_2}(x_2, \mu_F^2) + f_{\bar{q}}^{P_1}(x_1, \mu_F^2) f_q^{P_2}(x_2, \mu_F^2) \\ H_{gq}(x_1, x_2, \mu_F^2) &= f_g^{P_1}(x_1, \mu_F^2) \left(f_q^{P_2}(x_2, \mu_F^2) + f_{\bar{q}}^{P_2}(x_2, \mu_F^2) \right) \\ H_{gq}(x_1, x_2, \mu_F^2) &= H_{gq}(x_2, x_1, \mu_F^2). \end{aligned} \quad (2.20)$$

Then, the inclusive differential cross section may be expressed as

$$\begin{aligned}
 2S \frac{d\sigma^{P_1 P_2}}{dQ^2}(\tau, Q^2) &= \sum_q \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dz \delta(\tau - zx_1 x_2) \\
 &\quad [\mathcal{F}_{SM+VA,q} \mathcal{G}_{SM+VA,q} + \mathcal{F}_{SP,q} \mathcal{G}_{SP,q}] \\
 \mathcal{G}_{SM+VA,q} &\equiv H_{q\bar{q}}(x_1, x_2, \mu_F^2) \left\{ \Delta_{q\bar{q}}^{(0),SM}(z, Q^2, \mu_F^2) + a_s \Delta_{q\bar{q}}^{(1),SM}(z, Q^2, \mu_F^2) \right\} \\
 &\quad + \left\{ H_{qq}(x_1, x_2, \mu_F^2) + H_{gq}(x_1, x_2, \mu_F^2) \right\} a_s \Delta_{qq}^{(1),SM}(z, \mu_F^2) \\
 \mathcal{G}_{SP,q} &\equiv H_{q\bar{q}}(x_1, x_2, \mu_F^2) \left\{ \Delta_{q\bar{q}}^{(0),SP}(z, Q^2, \mu_F^2) + a_s \Delta_{q\bar{q}}^{(1),SP}(z, Q^2, \mu_F^2) \right\} \\
 &\quad + \left\{ H_{qq}(x_1, x_2, \mu_F^2) + H_{gq}(x_1, x_2, \mu_F^2) \right\} a_s \Delta_{qq}^{(1),SP}(z, \mu_F^2)
 \end{aligned} \tag{2.21}$$

with the constants $\mathcal{F}_{SM+VA,q}$ and \mathcal{F}_q^{SP} containing all the dependences on the coupling constants and propagators, namely,

$$\begin{aligned}
 \mathcal{F}_{SM+VA,q} &= \frac{4\alpha^2}{3} \left[\left\{ \frac{e_q^2}{Q^2} - 2e_q g_l^V g_q^V Z_Q \frac{Q^2 - M_Z^2}{Q^2} \right. \right. \\
 &\quad \left. \left. + \frac{1}{4} \left((g_l^R)^2 + (g_l^L)^2 \right) \left((g_q^R)^2 + (g_q^L)^2 \right) Z_Q \right\} \right. \\
 &\quad \left. + \frac{2}{\alpha\Lambda^2} \left\{ -e_q \sum_{i,j=L,R} \eta_{ij} + Z_Q(Q^2 - M_Z^2) \sum_{i,j=L,R} \eta_{ij} g_q^i g_l^j \right\} \right. \\
 &\quad \left. + \frac{Q^2}{\alpha^2 \Lambda^4} \sum_{i,j=L,R} |\eta_{ij}|^2 \right] \\
 \mathcal{F}_{SP,q} &= \frac{Q^2}{\Lambda^4} \sum_{i,j=L,R} |\xi_{ij}|^2 \\
 Z_Q &\equiv \frac{Q^2}{(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}
 \end{aligned} \tag{2.22}$$

For the vector-axial vector couplings, the results for the coefficient functions are analogous to the case of the SM [26], namely

$$\begin{aligned}
 \Delta_{q\bar{q}}^{(0),VA} &= \frac{2\pi}{N} \delta(1-z) \\
 \Delta_{q\bar{q}}^{(1),VA} &= \frac{8\pi C_F}{N} \left[\left\{ -4 + 2\zeta(2) \right\} \delta(1-z) - (1+z) \ln \frac{(1-z)^2}{z} - 2 \frac{\ln(z)}{1-z} + \right. \\
 &\quad \left. \left\{ \frac{2}{(1-z)_+} + \frac{3}{2} \delta(1-z) - (1+z) \right\} \ln \left(\frac{Q^2}{\mu_F^2} \right) + 4 \left(\frac{\ln(1-z)}{1-z} \right)_+ \right] \\
 \Delta_{q(\bar{q})g}^{(1),VA} &= \frac{2\pi}{N} T_F \left[2 \left\{ 1 - 2z + 2z^2 \right\} \ln \left(\frac{Q^2(1-z)^2}{z\mu_F^2} \right) + 1 + 6z - 7z^2 \right].
 \end{aligned} \tag{2.23}$$

For the scalar-pseudoscalar couplings, on the other hand, the LO coefficient function is given by

$$\Delta_{q\bar{q}}^{(0),SP} = \frac{2\pi}{N} \delta(1-z) \tag{2.24}$$

while at the next-to-leading order coefficient functions are

$$\begin{aligned}\Delta_{q\bar{q}}^{(1),SP} &= \frac{4\pi C_F}{N} \left[\left\{ -2 + 4\zeta(2) \right\} \delta(1-z) + 2(1-z) + 4(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ \right. \\ &\quad \left. + 2 \frac{1+z^2}{(1-z)_+} \ln \left(\frac{Q^2}{z\mu_F^2} \right) + 3\delta(1-z) \ln \left(\frac{Q^2}{\mu_F^2} \right) \right], \quad (2.25) \\ \Delta_{q(\bar{q})g}^{(1),SP} &= \frac{2\pi T_F}{N} \left[2 \left(1 - 2z + 2z^2 \right) \ln \left(\frac{Q^2(1-z)^2}{z\mu_F^2} \right) + (1-z)(7z-3) \right].\end{aligned}$$

The $SU(N)$ color factors in the above equations are

$$C_F = \frac{N^2 - 1}{2N}, \quad C_A = N, \quad T_F = \frac{1}{2}. \quad (2.26)$$

3. Differential cross sections with respect to dilepton rapidity

Having considered, in the previous section, the differential distributions with respect to the dilepton invariant mass, we now consider a second variable of interest, namely the rapidity of the pair. The latter can be expressed as

$$Y = \frac{1}{2} \log \left(\frac{p_2 \cdot q}{p_1 \cdot q} \right) = \frac{1}{2} \log \left(\frac{q_{CMH}^0 - q_{CMH}^3}{q_{CMH}^0 + q_{CMH}^3} \right) \quad (3.1)$$

with the second equality valid in the center of mass frame of the hadrons. Thus, the rapidity distribution may be computed simply by introducing the identity

$$\int dY \delta \left(Y - \frac{1}{2} \log \left(\frac{p_2 \cdot q}{p_1 \cdot q} \right) \right) = 1,$$

in eq. (2.2). This leads to

$$2S \frac{d\sigma^{P_1 P_2}}{dQ^2 dY}(\tau, Y, Q^2) = \frac{1}{2\pi} \sum_{j,j'=\gamma,Z,VA,SP} \tilde{P}_j(Q^2) \tilde{P}_{j'}^*(Q^2) \mathcal{L}_{jj'}(Q^2) \frac{dW_{jj'}^{P_1 P_2}}{dY}(\tau, Y, Q^2). \quad (3.2)$$

where the hadronic structure functions are given by

$$\begin{aligned}\frac{dW_{jj'}^{P_1 P_2}}{dY}(\tau, Y, Q^2) &= \sum_{a,b,j,j'} \int_0^1 dx_1 \int_0^1 dx_2 f_a^{P_1}(x_1) f_b^{P_2}(x_2) \int_0^1 dz \delta(\tau - zx_1 x_2) \\ &\quad \times \int dPS_{m+1} |\mathcal{M}^{ab \rightarrow jj'}|^2 T_{jj'}(q) \delta \left(Y - \frac{1}{2} \log \left(\frac{p_2 \cdot q}{p_1 \cdot q} \right) \right)\end{aligned} \quad (3.3)$$

We start with the leading order case which involves just the calculation of the square of the matrix element for the process $a(k_1) + b(k_2) \rightarrow j(q)$. The relevant phase space element corresponds to that for a $(0+1)$ -body final state, and

$$\begin{aligned}\int dPS_{0+1} \int dz \delta \left(Y - \frac{1}{2} \log \left(\frac{p_2 \cdot q}{p_1 \cdot q} \right) \right) \delta(\tau - zx_1 x_2) \\ = \frac{2\pi}{Q^2} \int dz \delta \left(Y - \frac{1}{2} \log \frac{x_1}{x_2} \right) \delta(1-z) \delta(\tau - zx_1 x_2).\end{aligned} \quad (3.4)$$

The integration over the rest of the variables is simplified, particularly in the context of the NLO corrections, by effecting a change of variables, namely

$$(Y, \tau) \longrightarrow (x_1^0, x_2^0) \equiv (\sqrt{\tau} e^Y, \sqrt{\tau} e^{-Y}) \quad (3.5)$$

Then it follows that

$$\begin{aligned} \frac{2\pi}{Q^2} \int dz \delta\left(Y - \frac{1}{2} \log\left(\frac{x_1}{x_2}\right)\right) \delta(1-z) \delta(\tau - zx_1x_2) |\mathcal{M}^{ab \rightarrow jj'}|^2 T_{jj'} \\ = \frac{2\pi}{Q^2} \delta(x_1 - x_1^0) \delta(x_2 - x_2^0) \left[|\mathcal{M}^{ab \rightarrow jj'}|^2 T_{jj'} \right]_{z=1}, \end{aligned} \quad (3.6)$$

rendering the remaining integrals trivial and thereby giving us the Born-level result for the Y -distribution. Having set the formalism, we may now calculate the next-to-leading-order contribution to the same. This involves the computation of matrix element squared for the processes $a(k_1) + b(k_2) \rightarrow j(q) + c(k)$. The $(1+1)$ -body phase space integration can be performed in the CM frame of the incoming partons wherein the particle momenta may be parametrised as

$$\begin{aligned} k_1 &= \frac{\sqrt{\hat{s}}}{2}(1, 0, \dots, 0, 1) \\ k_2 &= \frac{\sqrt{\hat{s}}}{2}(1, 0, \dots, 0, -1) \\ -q &= \frac{\sqrt{\hat{s}}}{2}(1+z, 0, \dots, -(1-z)\sin\theta, -(1-z)\cos\theta) \\ -k &= \frac{\sqrt{\hat{s}}}{2}(1-z, 0, \dots, (1-z)\sin\theta, (1-z)\cos\theta) \end{aligned}$$

Writing $\cos\theta = 2y - 1$, the two delta functions reduce to

$$\begin{aligned} \delta\left(Y - \frac{1}{2} \log\frac{p_2 \cdot q}{p_1 \cdot q}\right) &= \delta\left(Y - \frac{1}{2} \log\frac{x_1(1-y(1-z))}{x_2(z+y(1-z))}\right) \\ &= \frac{2x_1x_2x_1^0x_2^0(x_1x_2+x_1^0x_2^0)}{(x_1x_2-x_1^0x_2^0)(x_1x_2^0+x_1^0x_2)^2} \delta(y-y^*) \\ \delta(\tau - zx_1x_2) &= \frac{1}{x_1x_2} \delta(z-z^*) \end{aligned} \quad (3.7)$$

where,

$$y^* = \frac{x_2x_2^0(x_1+x_1^0)(x_1-x_1^0)}{(x_1x_2-x_1^0x_2^0)(x_1x_2^0+x_1^0x_2)}, \quad z^* = \frac{x_1^0x_2^0}{x_1x_2} \quad (3.8)$$

The above relations can be used to obtain

$$\begin{aligned} \int dPS_{1+1} \int dz \delta\left(Y - \frac{1}{2} \log\frac{p_2 \cdot q}{p_1 \cdot q}\right) \delta(\tau - zx_1x_2) |\mathcal{M}^{ab \rightarrow jj'}|^2 T_{jj'} \\ = \frac{1}{8\pi} \left(\frac{Q^2}{4\pi}\right)^{\epsilon/2} \frac{1}{\Gamma(1+\epsilon/2)} \frac{2x_1^0x_2^0(x_1x_2+x_1^0x_2^0)}{x_1x_2(x_1x_2^0+x_1^0x_2)^2} \left[|\mathcal{M}^{ab \rightarrow jj'}|^2 T_{jj'} \right]_{y=y^*, z=z^*} \\ \times \left(\frac{(x_1-x_1^0)(x_2-x_2^0)(x_1+x_1^0)(x_2+x_2^0)}{(x_1x_2^0+x_1^0x_2)^2} \right)^{\epsilon/2} \end{aligned} \quad (3.9)$$

To obtain the contribution to the Y distribution from real gluon emissions, we substitute eq. (3.9) in eq. (3.2). Similarly, the virtual corrections can be obtained using eq. (3.4) with one-loop corrected matrix elements. The soft singularities cancel after adding the real emission contributions and virtual corrections to the Born process. The remaining collinear divergences are removed by mass factorization, or, in other words, by replacing the bare parton distribution with the renormalized ones using the Alteralli-Parisi kernels as follows

$$f_a^P(z) = \sum_b \Gamma_{ab}^{-1} \otimes f_b^P(z, \mu_F^2), \quad (3.10)$$

which implies

$$\begin{aligned} f_q^P(z) &= f_q^P(z, \mu_F^2) - \frac{a_s}{\epsilon} \left[\Gamma_{qq}^{(1)} \otimes f_q^P(z, \mu_F^2) + \Gamma_{qg}^{(1)} \otimes f_g^P(z, \mu_F^2) \right] \\ f_{\bar{q}}^P(z) &= f_{\bar{q}}^P(z, \mu_F^2) - \frac{a_s}{\epsilon} \left[\Gamma_{\bar{q}\bar{q}}^{(1)} \otimes f_{\bar{q}}^P(z, \mu_F^2) + \Gamma_{\bar{q}g}^{(1)} \otimes f_g^P(z, \mu_F^2) \right] \\ f_g^P(z) &= f_g^P(z, \mu_F^2) - \frac{a_s n_f}{\epsilon} \left[\Gamma_{gq}^{(1)} \otimes f_q^P(z, \mu_F^2) + \Gamma_{g\bar{q}}^{(1)} \otimes f_{\bar{q}}^P(z, \mu_F^2) \right. \\ &\quad \left. + \Gamma_{gg}^{(1)} \otimes f_g^P(z, \mu_F^2) \right]. \end{aligned} \quad (3.11)$$

Thus, we finally have, for the one-loop corrected distributions in the dilepton pair rapidity,

$$\begin{aligned} 2S \frac{d\sigma}{dQ^2 dY}(\tau, Y, Q^2) &= \sum_{i=q} \mathcal{F}_{SM+VA,q} \left[D_{q\bar{q}}^{SM}(x_1^0, x_2^0, \mu_F^2) \right. \\ &\quad \left. + D_{qg}^{SM}(x_1^0, x_2^0, \mu_F^2) + D_{gq}^{SM}(x_1^0, x_2^0, \mu_F^2) \right] \\ &+ \sum_{i=q} \mathcal{F}_{SP,q} \left[D_{q\bar{q}}^{SP}(x_1^0, x_2^0, \mu_F^2) \right. \\ &\quad \left. + D_{qg}^{SP}(x_1^0, x_2^0, \mu_F^2) + D_{gq}^{SP}(x_1^0, x_2^0, \mu_F^2) \right] \end{aligned} \quad (3.12)$$

with the constants $\mathcal{F}_{SM+VA,q}$ and $\mathcal{F}_{SP,q}$ as in eq. (2.22). The functions D can be split conveniently into the Born-approximation piece and the NLO corrections, viz

$$D_{ab}^\eta(x_1^0, x_2^0, \mu_F^2) = D_{ab}^{\eta,(0)}(x_1^0, x_2^0, \mu_F^2) + a_s D_{ab}^{\eta,(1)}(x_1^0, x_2^0, \mu_F^2) \quad (\eta = SM, VA, SP). \quad (3.13)$$

Once again, the analytical expressions for the VA -type contact interactions are the same as those obtained within the SM and can be found in ref. [26]. As for the SP -type interactions, while the leading-order expression is simple

$$D_{q\bar{q}}^{SP,(0)}(x_1^0, x_2^0, \mu_F^2) = \frac{\pi}{N} H_{q\bar{q}}(x_1^0, x_2^0, \mu_F^2) \quad (3.14)$$

the NLO results are more complicated. Defining, for convenience, certain constants

$$\begin{aligned} \kappa_{a_1} &= \ln \frac{2Q^2(1-x_2^0)(x_1-x_1^0)}{\mu_F^2(x_1+x_1^0)x_2^0} & \kappa_{b_1} &= \ln \frac{Q^2(1-x_2^0)(x_1-x_1^0)}{\mu_F^2 x_1^0 x_2^0} \\ \kappa_{c_1} &= \ln \frac{2x_1^0}{x_1+x_1^0} & \kappa_{12} &= \ln \frac{(1-x_1^0)(1-x_2^0)}{x_1^0 x_2^0} \end{aligned} \quad (3.15)$$

we have

$$\begin{aligned}
D_{q\bar{q}}^{SP,(1)}(x_1^0, x_2^0, \mu_F^2) &= \left(\frac{2\pi C_F}{N} \right) \left\{ \varphi_0^{g\bar{q}} + \int dx_1 \varphi_1^{g\bar{q}} + \int dx_1 dx_2 \varphi_2^{g\bar{q}} \right\} + (1 \leftrightarrow 2) \\
\varphi_0^{g\bar{q}} &= \frac{1}{2} H_{q\bar{q}}(x_1^0, x_2^0, \mu_F^2) \left(-2 + \kappa_{12}^2 + 6\zeta(2) + (3 + 2\kappa_{12}) \ln \frac{Q^2}{\mu_F^2} \right) \\
\varphi_1^{g\bar{q}} &= \frac{2\kappa_{b1}}{x_1 - x_1^0} H_{q\bar{q},1}(x_1, x_2^0, \mu_F^2) \\
&\quad + H_{q\bar{q}}(x_1, x_2^0, \mu_F^2) \left(\frac{1 - \kappa_{a1}}{x_1} + \frac{2\kappa_{c1}}{x_1 - x_1^0} - \frac{1 + \kappa_{a1}}{x_1^2} x_1^0 \right) \\
\varphi_2^{g\bar{q}} &= \frac{H_{q\bar{q},12}(x_1, x_2, \mu_F^2)}{(x_1 - x_1^0)(x_2 - x_2^0)} - \frac{x_2 + x_2^0}{(x_1 - x_1^0)x_2^2} H_{q\bar{q},1}(x_1, x_2, \mu_F^2) \\
&\quad + \frac{H_{q\bar{q}}(x_1, x_2, \mu_F^2)}{2x_1^2 x_2^2} \left((x_1 + x_1^0)(x_2 + x_2^0) + \frac{x_1^2 x_2^2 + x_1^0 x_2^0}{(x_1 + x_1^0)(x_2 + x_2^0)} \right)
\end{aligned} \tag{3.16}$$

and

$$\begin{aligned}
D_{gq}^{SP,(1)}(x_1^0, x_2^0, \mu_F^2) &= \frac{2\pi T_f}{N} \int \frac{dx_1}{x_1^3} \\
&\quad \times \left[\varphi_1^{g\bar{q}} + \int dx_2 \left\{ \varphi_2^{g\bar{q}} - \frac{\varphi_3^{g\bar{q}} H_{gq}(x_1, x_2, \mu_F^2)}{x_2^2 (x_2 + x_2^0) (x_1 x_2^0 + x_2 x_1^0)^3} \right\} \right] \\
\varphi_1^{g\bar{q}} &= H_{gq}(x_1, x_2^0, \mu_F^2) \left(2x_1^0(x_1 - x_1^0) + \kappa_{a1} (x_1^0 + (x_1 - x_1^0)^2) \right) \\
\varphi_2^{g\bar{q}} &= \frac{H_{gq,2}(x_1, x_2, \mu_F^2)}{x_2 - x_2^0} \left(x_1^0 + (x_1 - x_1^0)^2 \right) \\
\varphi_3^{g\bar{q}} &= -x_1^5 x_2^2 x_2^0 + x_1^4 x_1^0 x_2^2 x_2^0 (3x_2 + 4x_2^0) \\
&\quad + x_1^3 x_1^0 x_2 x_2^0 (3x_2^3 + 2x_2^0) + 2x_1^0 x_2^2 (x_2^3 + 2x_2^2 x_2^0 + 2x_2 x_2^0 + 2x_2^0) \\
&\quad + 2x_1 x_1^0 x_2 (-x_2^4 + x_2^3 x_2^0 + 4x_2^2 x_2^0 + 2x_2 x_2^0 + 2x_2^0) \\
&\quad + x_1^2 x_1^0 (x_2^5 - 4x_2^4 x_2^0 - 4x_2^3 x_2^0 + 2x_2^2 x_2^0 + 2x_2 x_2^0 + 2x_2^0)
\end{aligned} \tag{3.17}$$

with

$$D_{gq}^{SP,1}(x_1^0, x_2^0, \mu_F^2) = D_{gq}^{SP,(1)}(x_1^0, x_2^0, \mu_F^2)|_{(1 \leftrightarrow 2)} \tag{3.18}$$

and we have used the following notations,

$$\begin{aligned}
H_{ab,12}(x_1, x_2, \mu_F^2) &= H_{ab}(x_1, x_2, \mu_F^2) - H_{ab}(x_1^0, x_2, \mu_F^2) - H_{ab}(x_1, x_2^0, \mu_F^2) \\
&\quad + H_{ab}(x_1^0, x_2^0, \mu_F^2) \\
H_{ab,1}(x_1, x_2, \mu_F^2) &= H_{ab}(x_1, x_2, \mu_F^2) - H_{ab}(x_1^0, x_2, \mu_F^2) \\
H_{ab,2}(x_1, x_2, \mu_F^2) &= H_{ab}(x_1, x_2, \mu_F^2) - H_{ab}(x_1, x_2^0, \mu_F^2).
\end{aligned} \tag{3.19}$$

4. Results and discussion

We now present numerical results relevant for the Run II of the Tevatron ($\sqrt{S} = 1.96$ TeV) as well as for the LHC ($\sqrt{S} = 14$ TeV). Although our goal, namely the calculation of the NLO QCD corrections, would be quite independent of the value of the contact interaction scale Λ , for definiteness we choose $\Lambda = 6$ (20) TeV for the Tevatron (LHC). Furthermore, in presenting our results, we shall consider only one of the couplings η_{AB}^q and ξ_{AB}^q to be non-zero and of unit strength.

For the sake of convenience, we parametrize the cross section as

$$\begin{aligned} \sigma &= \sigma_{\text{SM}} + \sigma_{\text{intf}} + \sigma_{\eta^2} && \text{(for the VA case)} \\ \sigma &= \sigma_{\text{SM}} + \sigma_{\xi^2} && \text{(for the SP case)} \end{aligned} \tag{4.1}$$

and similarly for the differential cross sections. This has the advantage in that the total cross sections, for an arbitrary value of Λ can be easily reconstructed.

4.1 The invariant mass distribution for the dilepton pair

In figure 1, we present the invariant mass distributions corresponding to the *VA* case. Note that the σ_{η^2} piece (and, similarly, the σ_{ξ^2} piece) depends only on the identity of the quark q taking part in the contact interaction and is independent of the chirality structure of the coupling. The interference term, on the other hand, does depend on the chirality structure as figure 1(a) amply demonstrates. As for σ_{SM} , the rapid decrease in cross section with M is reflective of both the s^{-1} fall of the parton-level cross section as well as the rapid fall in parton distribution functions at higher momentum fractions. That the interference terms do not fall as fast is a consequence of the higher dimensional nature of the contact interaction lagrangian. This, naturally, is even more evident for the σ_{η^2} (σ_{ξ^2}) piece. Consequently, at high M values, the contact interaction contribution dominates over the SM piece. For the LHC, this dominance occurs at a larger M value as compared to the case of the Tevatron precisely because we have chosen to work with a much larger value of Λ for the former environment. And, expectedly, for identical couplings, the cross section due to a $u\bar{u}$ initial state dominates that originating from a $d\bar{d}$ initial state. In figure 1, we have chosen to limit ourselves only to these two initial states as the cross sections corresponding to the heavier quarks would be suppressed even further (note though that the experimental bounds on Λ is relaxed too for such cases).

We should clarify, at this stage, that, in calculating the NLO cross sections shown in figure 1, we have made a specific choice of the renormalization scale μ_R and the factorization scale μ_F , namely,

$$\mu_R = \mu_F = M \equiv \sqrt{Q^2}.$$

Postponing, until later, a discussion of the dependence on the scale choice, we may define now a invariant mass-dependent K -factor, namely

$$K_M^q = \left[\frac{d\sigma_{LO}(M)}{dM} \right]^{-1} \left[\frac{d\sigma_{NLO}(M)}{dM} \right], \tag{4.2}$$

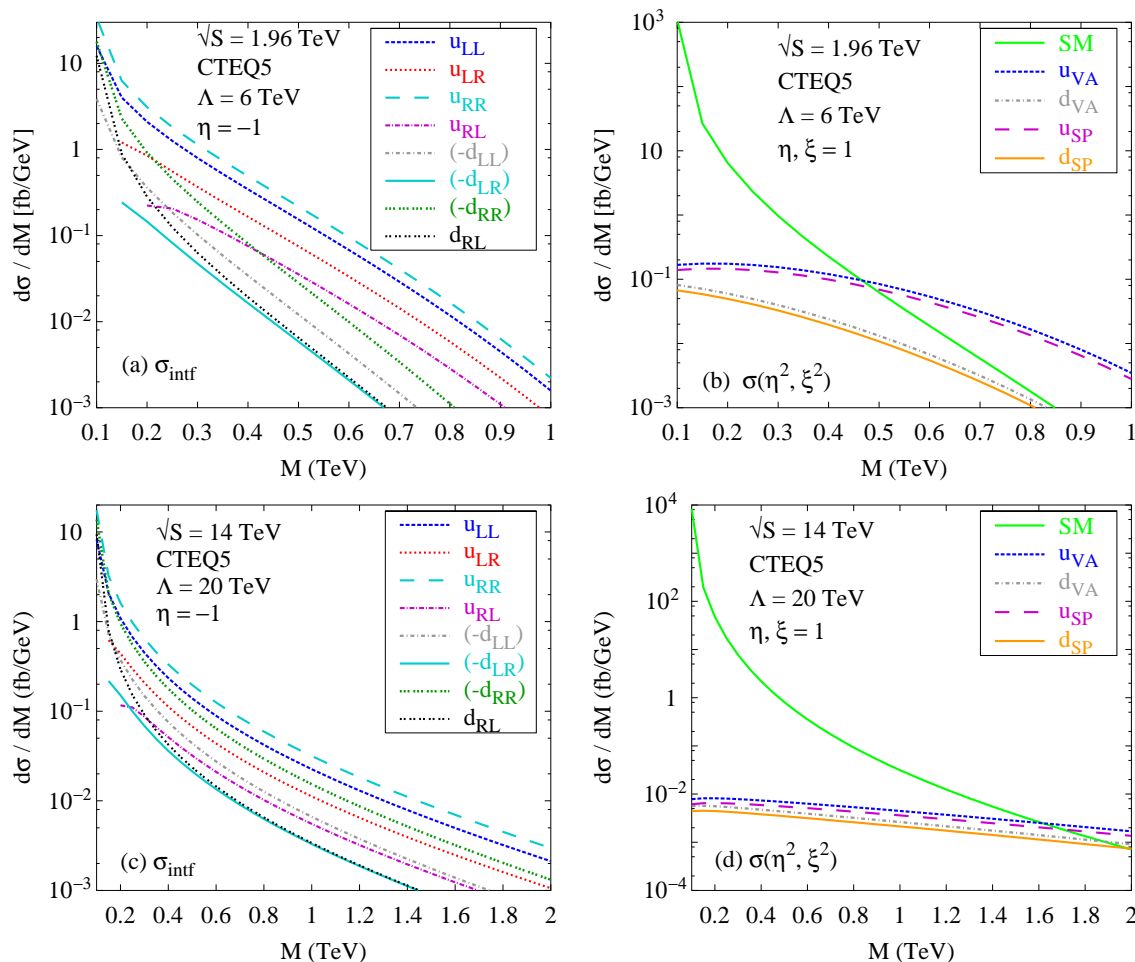


Figure 1: The differential inclusive dilepton production cross-sections (at NLO) for the contact interaction terms. In each case, only one coupling (η, ξ) is assumed to be non-zero and of unit size. Also shown, for comparison, is the total SM contribution. The top and bottom panels refer to the Tevatron ($\sqrt{S} = 1.96$ TeV and $\Lambda = 6$ TeV) and the LHC ($\sqrt{S} = 14$ TeV and $\Lambda = 20$ TeV) respectively. The right and left panels refer respectively to the pure contact interaction term and the interference with the SM.

where q refers to the identity of the quark, and the LO (NLO) cross sections are computed by convoluting the corresponding parton-level cross sections with the LO (NLO) parton distribution functions. In figure 2, we exhibit the variation of K_M with M for different choices of q .

As derived in the previous section, and as already evinced in figure 1, the fractional correction depends only on the spin structure of the vertex, and not on the chirality. Thus, for a given quark, the K -factor would depend on whether the interaction is of SP or VA type, but within each class, the chirality structure (namely whether it is LL , RR , LR or RL type) is quite irrelevant. The last statement also implies that, for the VA -type interaction, the K -factor would be exactly the same as in the SM , as far as the particular quark initial state is concerned. Numerically, this feature is displayed in figure 2. Of course,

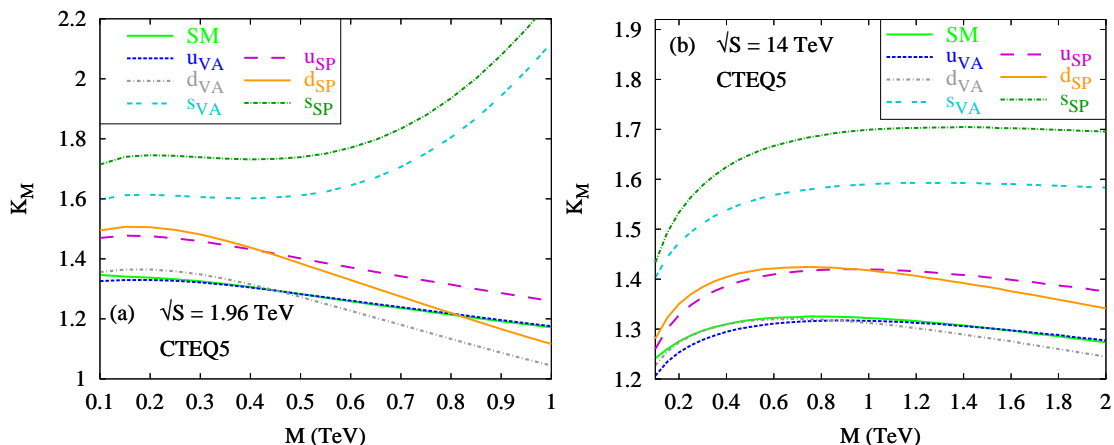


Figure 2: The K -factors for the differential (in dilepton invariant mass) cross-section for (a) the Tevatron Run II and (b) the LHC. For the contact interactions, the K -factors are independent of the chirality structure of the operators, but depend on whether they are of the VA or the SP type.

the K -factor *does* depend on the identity of q . As can be expected, $K_M^u(VA)$ and $K_M^d(VA)$ are relatively close to each other and, in turn, to $K_M(SM)$. In fact, to a large measure, $K_M(SM)$ is but the weighted average of the other two, with the relative strengths being determined by the quark fluxes. That these K -factors fall monotonically with M for the case of the Tevatron and not so for the LHC is understandable in the light of the fact that, the former is a $\bar{p}p$ machine, while the latter is a pp one. As for $K_M^s(VA)$, the steep rise at large M values is but a reflection of the dominance of the Compton-like subprocess ($sg \rightarrow \ell^+\ell^-s$ and $\bar{s}g \rightarrow \ell^+\ell^-\bar{s}$) owing to the larger flux of gluons as compared to s/\bar{s} , especially for large momentum fractions.

The results for the SP -type interactions are qualitatively similar, though quantitatively the K -factors are significantly larger than those for the VA -type interaction (or the SM). The numerical differences are but consequences of the the structures of the respective matrix elements. On closer inspection, $K_M^q(SP)$, for a given M , turns out to be the same as that for resonance production of a scalar/pseudoscalar of mass M [29].

4.2 The rapidity distributions

We now turn to the distribution in a different kinematical variable, namely Y , *the rapidity of the lepton pair*¹. However, rather than look at $d\sigma/dY$ itself, we shall rather consider on $d^2\sigma/dM dY$, for this allows us to accentuate the effect of the contact interactions by concentrating on a suitable M range.

At the LO, the variable Y is just a measure of the boost of the partonic center of mass with respect to the laboratory frame. At the NLO, one has to consider the effect of the initial state radiation as well. In either case, it is easy to see that $d^2\sigma/dM dY$ (and, hence, $d\sigma/dY$) is an even function of Y . In figure 3, we exhibit the dependence of $d^2\sigma/dM dY$ on Y , for a fixed value of M . As expected, for a large enough value of the latter, the effect of

¹This should be distinguished from the rapidity of an individual lepton.

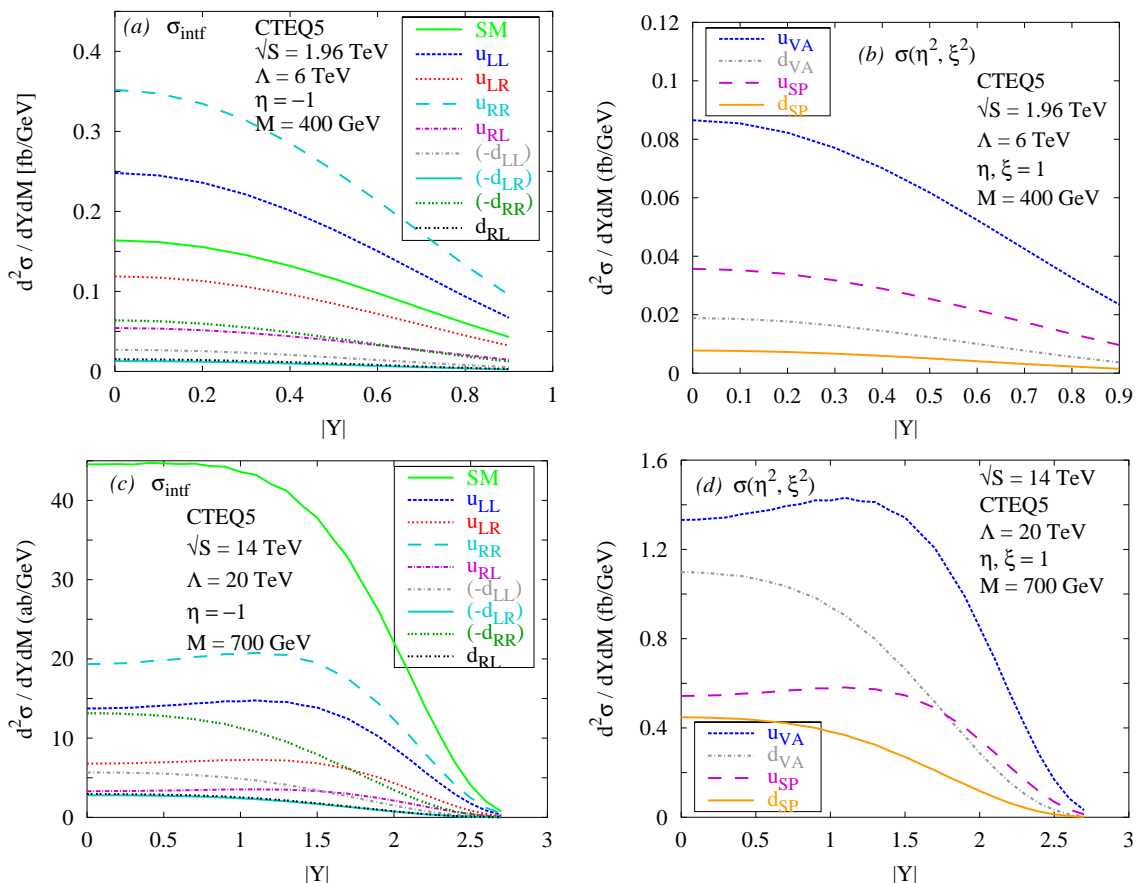


Figure 3: As in figure 1, but for the double differential (in rapidity Y and mass M) distribution instead.

the contact interaction is clearly discernible and especially for the central rapidity region. Note that the contact interaction cross sections are significantly flatter in Y than the SM contribution. Once again, this is a reflection of the structure of the new physics matrix element as compared to that due to γ/Z exchange.

Analogous to K_M defined in the previous subsection, one may now define a Y -dependent K -factor of the form defined as

$$K_Y \equiv \left[\frac{d\sigma_{LO}(M, Y)}{dM dY} \right]^{-1} \left[\frac{d\sigma_{NLO}(M, Y)}{dM dY} \right], \quad (4.3)$$

and we plot this quantity as a function of Y in figure 4. The results are quite reminiscent of those for K_M (as displayed in figure 2). It is noteworthy that, for the LHC, K_Y^s shows a large upward swing at large Y , whereas K_M^s had seemed better behaved. The reason is not difficult to fathom. For the range of M spanned in figure 2(b), the cross section integral typically samples relatively moderate values of the Bjorken- x as compared to the higher- $M_{\ell\ell}$ regime for the Tevatron case (figure 2(a)). On the other hand, a phase space point such as $(M = 700 \text{ GeV}, |Y| = 2.5)$ necessarily pushes one to larger momentum fractions

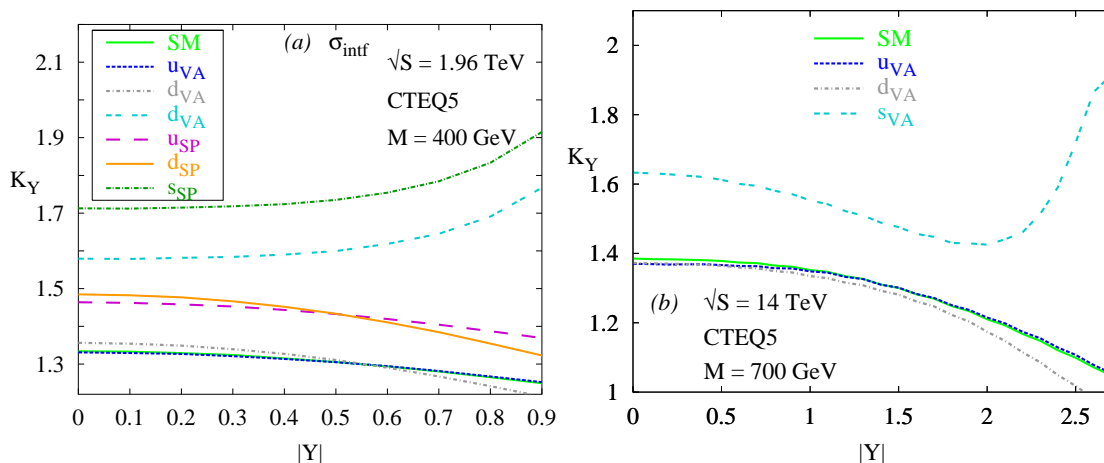


Figure 4: As in figure2, but for the double differential (in rapidity Y and mass M) distribution instead.

for the partons and thus, once again, it is the ratio of the strange-quark flux to that of the gluon that causes the upward turn in figure 4(b).

4.3 The choice of scale

Until now, we have chosen each of the factorization scale μ_F (relevant to both the LO as well as NLO calculations) and the renormalization scale μ_R (relevant only for the NLO case) to be the same as the dilepton invariant mass M . As is well known, this choice is arbitrary and there is no theoretical guideline for making such a choice. Maintaining, for reasons of simplicity, $\mu_R = \mu_F$, we now examine the dependence of our calculations on this choice. To quantify the scale dependence of our result, we define ratios R_M and R_Y

$$\begin{aligned}
 R_M^I(\mu_F) &\equiv \left[\frac{d\sigma_I(M, \mu_F = M)}{dM} \right]^{-1} \left[\frac{d\sigma_I(M, \mu_F)}{dM} \right], \\
 R_Y^I(\mu_F) &\equiv \left[\frac{d\sigma_I(M, Y, \mu_F = M)}{dM dY} \right]^{-1} \left[\frac{d\sigma_I(M, Y, \mu_F)}{dM dY} \right],
 \end{aligned}
 \tag{4.4}$$

where $I = LO, NLO$. A value of $R_{M,Y}^I(\mu_F)$ close to unity would then signify a low sensitivity to the choice of scale and hence a more robust result.

In figure 5, we display the above ratios for the case of the LHC and the VA interactions. Note that the variation of the cross section with the factorization scale is relatively small. Furthermore, the variation reduces significantly as one goes from the LO to the NLO case. This immediately points to the increased robustness of the prediction on inclusion of the corrections, and lends hope that the remaining scale ambiguity can, presumably, be reduced by adding still higher order corrections. Note that, at the leading order, these ratios are independent of the dynamics and reflect only the effect of the choice of the factorization scale on the parton densities. In other words,

$$R_M^{LO}(SP) = R_M^{LO}(VA), \quad R_Y^{LO}(SP) = R_Y^{LO}(VA).$$

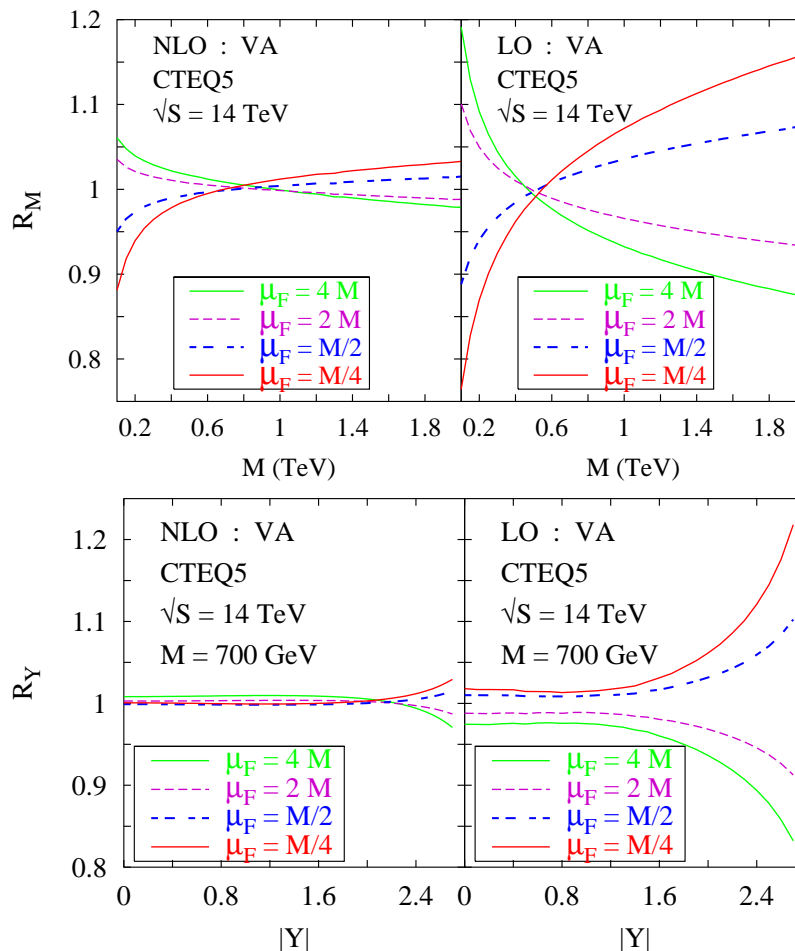


Figure 5: The K -factors for the differential (in dilepton invariant mass) cross-section for (a) the Tevatron Run II and (b) the LHC. For the contact interactions, the K -factors are independent of the chirality structure of the operators, but depend on whether they are of the VA or the SP type.

At the next-to-leading order, the dynamics does play a role. However, the differences between the R -ratios for the SP and VA cases are too small to be noticeable on the scale of figure 5. The results are similar for the case of the Tevatron as well.

5. Conclusions

To summarize, we have performed a systematic calculation of the next-to-leading order QCD corrections for the Drell-Yan process in theories with contact interactions. Contrary to naive expectations, we demonstrate explicitly that the QCD corrections are meaningful and reliable in the sense that no undetermined parameters need be introduced.

We have analyzed both the invariant mass distribution and the rapidity distributions for the dilepton pair at either of the Tevatron and the LHC. The enhancements over the LO expectations are found to be quite significant. The corresponding K -factors are presented in a form suitable for use in experimental analyzes.

For the VA -type interactions, the analytical structure of the corrections are similar to those for the SM. However, a significant dependence on the flavour structure is found and needs to be carefully accounted for in obtaining any experimental bounds. For the SP -type interaction, not only are the analytical results quite different, but the consequent K -factors are typically larger than those within the SM.

Finally, we have investigated the sensitivity of our results to both the factorization and renormalization scales. As expected, we find such dependences to be greatly reduced for the case of the NLO results as compared to that for the LO case. This indicates the robustness of the calculations.

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